

Comparisons between Huber's and Newton's Multiple Regression Models for Stem Biomass Estimation

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ABSTRACT

This paper employed the technique of Multiple Regression (MR) in estimating the tree stem volume of *Roystonea regia* (*R. regia*) based on two volumetric equations, namely, the Huber's and Newton's formulae. Variables considered for data mensuration were stem height (or bole), tree height, diameter at breast height, diameter at middle and diameter at top of the stem before the crown. Correlation coefficient and normality tests were done to screen and select possible variables with their interactions. Transformations were done for normality and variables with Pearson correlation coefficient values greater than 0.95 were eliminated to reduce multicollinearity. All selected models were examined using parameters tests: Global test, Coefficient test and the Wald test. The Wald test was carried out to justify the elimination of the insignificant variables. The eight criteria model selection (8SC) process was done to obtain the best regression model without effects of multicollinearity and insignificant variables. Major contributors to the best Multiple Regression (MR) model were from tree height and diameter at the middle of the stem, while significant contributions were from the bole (h) and diameters at breast height (Dbh) and the top, Dt .

Keywords: stem volume, correlation coefficient, multicollinearity, 8SC, best multiple regression.

1. INTRODUCTION TO BIOMASS ESTIMATION

The global atmospheric CO₂ build-up is partially due to deforestation. Alternatively, afforestation appears to be one of the feasible methods of reducing the concentration of CO₂ from the atmosphere. It uses solar energy and allows an economic fixation of CO₂ from the atmosphere which does not depend on concentrated CO₂ streams. Control of dispersed sources of CO₂ is also taken into account by photosynthetic extraction of CO₂ from the atmosphere.

Trees properly used in a landscape could increase property values by as much as 20 percent, besides providing food and shelter for birds and

urban wildlife. Planted strategically, the right shade trees could further reduce building cooling costs by as much as 50 percent. Burns (2006) also discovered that trees were found to reduce the temperature of streets and parking lots by 8 to 10 degrees in the summer, hence making paved surfaces last longer without repairs. They would also improve air quality by trapping dust, absorbing air pollutants and converting carbon monoxide to oxygen which is essential towards mankind environment.

Hoffman and Usoltsev (2002) studied on the tree–crown biomass estimation in the forest species of the Ural and Kazakhstan, had stated that there were two separate most economical and relatively precise regressions, one for broad-leaved while the other for coniferous species, each only use stem diameter at the lowest point of the crown, *Db*. Approximation for coniferous foliage was found to have improved considerably by allowing parallel regressions, inclusive of mean diameter increment and diameter at breast height as predictors; however, tree age being less influential than its mean increment.

Wang (2006) also developed an allometric equation relating component biomass to independent variables, such as, diameter at breast height (*Dbh*) and tree height (*TH*) for 10 co-occurring tree species in China's temperate forests, by using simple linear regression, and then executed the PROC GLM procedure in SAS for analysis. The foliage biomass was found to be more variable than other biomass components, both across and within tree species.

Estimation of crown characters and leaf biomass from leaf litter in a Malaysian canopy species, *Elateriospermum tapos* (*Euphorbiaceae*) was also studied by Osada *et al.* (2003). Estimated values were found to be similar to the values estimated from the allometric equation which used parameters such as, the diameter at breast height and the overall tree height. Forest productivity was evaluated and the characteristics in various forests were studied using litter trap method which in turn, estimated by the non-linear least square regression.

The increasing desire for total tree utilization and the need to express yield in terms of weight rather volume had stimulated studies of biomass production by Fuwape *et al.* (2001). Even-aged stands of *Gmelina arborea* and *Nauclea diderrichii* in Nigeria were studied to obtain the biomass equations and estimation of both of the species. *Nauclea diderrichii*, an indigenous species was found to thrive well in plantations. Onyekwelu (2004) had assessed the above ground biomass production of

even-aged stands of *Gmelina* by non-destructive method and recommended that models incorporating *Dbh* only to be used for estimating the biomass production.

Noraini *et al.* (2008) had also studied the stem biomass estimation of *Cinnamomum iners* using the multiple regression technique. Stem height (bole) and diameters at breast height, middle and the top were found to be significant contributing factors and should be incorporated into the regression models.

2. SITE DESCRIPTION AND MATERIALS

The data set were measured from a commonly grown tree, *Roystonea regia* (*R. regia*), found in Universiti Malaysia Sabah main campus in Sabah. Located on a 999-acres piece of land along Sepanggar Bay in Kota Kinabalu, at latitude 6° 00' and longitude 116° 04'. The region has a mean annual rainfall around 2000-2499 mm with a relative humidity of 81.2±0.3%. The mean annual temperature is 27.2±0.1°C and its seasonal rain extends from October to January. Only data from trees, located along the steps starting from the Chancellery Hall up towards the Chancellery, were measured and collected. Variables were measured using clinometers and fibreglass girth tape. The clinometer is used to measure the height of a tree while the fibreglass tape measures the diameter indirectly by wrapping round the tree to measure the circumference in a perpendicular plane to the stem axis, and its value divided by π (π) to estimate the diameter.

3. ROYSTONEA REGIA, PHYSICAL AND FUNCTIONAL USES

The common name for *Roystonea regia* (*R. regia*) is the Royal Palma or *R. regia*, in short. Belonging to the palm family of Palmae or Areaceae, it is a native of Cuba, but now have naturalized in Hawaii, Florida and most parts of the world with a subtropical moist and subtropical wet life zones. Its growth can be rapid to a massive height of 15.0-34.5 meters with 61 cm in diameter, and symmetrical with a smoothly sculpted trunk, lending a distinctive air to parkways and boulevards. Primarily valued as an ornamental tree, hence being used in urban landscaping, its fruits are also a source of oil. In some parts of the Caribbean, like Cuba, the

leaf-bases are used for roof thatches, and the trees for timber, livestock feed and palmito which is the edible terminal bud of the heart-of-palm.

The procedures of measurement done for the *R. regia* were as follows. Firstly, the height of stem (bole) for each palm tree was measured from the land at ground level up to where the colours of the tree stem started to differ by using clinometers. The diameters of the stem measured were at breast height, middle and top of the stem using a diameter girth tape. The main variables of the mensuration data would include the followings:

- h = stem height of tree (bole)
- At = area at the top of stem
- Am = area at the middle of stem
- Ab = Area at the base of stem with Db as the diameter at the base
- Dbh = diameter at breast height
- Dt = the diameter at the top
- Dm = diameter at the middle or halfway along the log
- Db = the diameter at the base

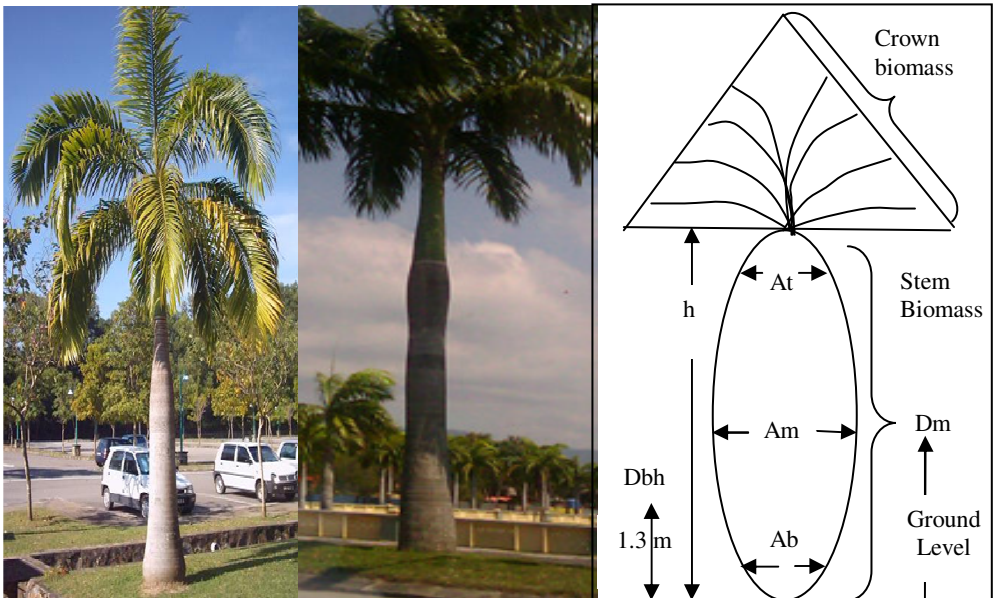


Figure 1: *R. regia* grown around the main campus (UMS).

Figure 2: Schematic diagram of *R. regia*.

4. BIOMASS EQUATIONS

Two volumetric formulae were used to calculate the stem volume of a tree. They were the Huber's and Newton's formulae which used different measurable variables in their equations. Figure 2 illustrates the schematic diagram of *R.regia* and the variables measured for field data collection.

a) Newton's Formula

The basal area of every mean tree in a tree population could be calculated using the formula, $Ab = \pi(Dbh)^2 / 4$, where Ab is the area at the base, and Dbh , the diameter at breast height. Stem volumes of the mean tree were then estimated using the Newton's formula (Fuwape *et al.*(2001)), $VN = h(Ab + 4Am + At) / 6$, where VN would be the volume using Newton's formula (m^3), h as the stem height (bole), and Ab , Am and At were the areas at the base, middle and top, respectively.

b) Huber's Formula

The main stem, up to merchantable height, is theoretically divided into a number of (mostly) standard length sections. The standard length is normally 3m (~10 feet). The exception to the standard section is the odd log - a section less than the standard length that fits between the last standard section and the merchantable height. These sections are assumed to be second degree paraboloids in shape. The bole from the merchantable height to the tip is assumed to be conoid in shape. The Huber's formula is based on the assumption that the sections are second degree paraboloids. However, this may not be appropriate for the bottom or base log-which is often neiloid. Huber's formula will underestimate the volume of a neiloid. However, this underestimate will be small if the difference in diameter between the bottom and the top of the section is small (i.e. small rate of taper or small sectional length). Thus, sections measured smaller than 3 meters may be necessary to avoid bias. Error in the standard sectional estimate of volume may also be introduced where the tip is not like a conoid. However, the volume in the tip is relatively small, so this error is likely to be unimportant (Brack (2006)).

The Huber's formula was used to calculate the volume of the standard sections and the odd log. It was given by: $VH = h \times S_h$. Taking the cross-sectional area (cm^2) halfway along the log as S_h , then $VH = h \times \pi \times (Dm^2) / 40000$, where VH was the volume using the Huber's

formula (m^3), h was the stem height or bole (m), and Dm denoted the diameter (cm) halfway along the log (Brack (2006)).

5. METHODOLOGY

Data Transformations of Normality

The data variables were measured from 72 trees non-destructively. For simplicity, the definitions of the variables are given in Table 1.

TABLE 1: Definition of Variables

Variable Name	Definition
$VN; VH$	Volume of stem (m^3):N-Newton's; H-Huber's
h	Stem height (bole) from the ground to the top before crown (m)
TH	Tree height from ground to the peak of tree (m)
Dbh	Diameter at breast height (m for Newton's; cm for Huber's)
Dm	Diameter at middle of trunk (m for Newton's; cm for Huber's)
Dt	Diameter at top of trunk (m for Newton's; cm for Huber's)

At the preliminary stage as with most environmental data, normalities of variables were a problem that had to be addressed. Hence, appropriate transformations were necessary. Since the data were not normally distributed (Lind *et al.* 2005), data transformations were done for normality. The variables were then tested for their normality distribution based on Kolmogorov-Smirnov statistics with Lilliefors significance level of more than 0.05, since the sample size was large ($n > 50$). This test was based on the null hypothesis that the data set was normally distributed.

The objective of this paper is to compare models using multiple regressions (*MR*) technique based on the two volumetric biomass equations. The *MR* models were made of a dependent variable, V , the stem volume and five independent variables, taken from field data mensuration. The phases in the methodology involved in modelling the *MR* models would be illustrated further in the subsequent subsections on the model-building procedures.

6. PHASE 1: FORMATION OF ALL POSSIBLE MODELS

The number of possible models was obtained by using the formula $\sum_{j=1}^{q=5} j({}^qC_j)$, where q is the number of single independent variables.

For five single independent variables and together with its combinations of interactions, a total of 80 possible models (as shown in Table 2 and Appendix A) could be obtained, before any regression procedures were done.

TABLE 2: Number of Possible Models before Regression Procedures.

Number of Variables	Individual	Interactions				Total
		First Order	Second Order	Third Order	Fourth Order	
1	${}^5C_1 = 5$	NA	NA	NA	NA	5
2	${}^5C_2 = 10$	10	NA	NA	NA	20
3	${}^5C_3 = 10$	10	10	NA	NA	30
4	${}^5C_4 = 5$	5	5	5	NA	20
5	${}^5C_5 = 1$	1	1	1	1	5
Total	31	26	16	6	1	80
Models	M1-M31	M32-M57	M58-M73	M74-M79	M80	

For example, taking the definitions of variables from Table 1, one of the possible models (M36) would be given by:

$$Y_{36} = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \beta_{12} X_{12} + \beta_{13} X_{13} + \beta_{23} X_{23} + \beta_{123} X_{123} + u \quad (1)$$

Each of these possible models could be written in a general form as in equation (2).

$$V = \Omega_0 + \Omega_1 W_1 + \Omega_2 W_2 + \dots + \Omega_k W_k + u, \quad (2)$$

where W is an independent variable which might come from one of these types of variables, namely, single independent, interactive, generated, transformed or even dummy variables (Peck *et al.* (2008)).

7. PHASE 2: PROCEDURES IDENTIFYING SELECTED MODELS

a) Multicollinearity Removal

The field data variables were initially in Microsoft Excel together with its all possible interactions. The existence of an exact linear relationship between the variables was examined from the Pearson Coefficient Correlation matrix in SPSS. The correlation between the variables was then investigated based on the Pearson Correlation Coefficient. Any absolute values of highly correlated variables ($|r| > 0.95$ was taken) were eliminated or excluded from the model so as to reduce the effects of multicollinearity (Ramanathan (2002)). The number of case types due to multicollinearity (Noraini *et al.* (2011)), and the number of variables removed due to multicollinearity (Zainodin *et al.* (2011)), would be denoted by the letter 'b', which came after the parent model, say *Ma*. The model then would be known as *Ma.b*. These models without multicollinearity were then examined by eliminating the insignificant variables.

b) Backward Elimination of Insignificant Variables

The backward elimination procedures began with the full model of all affecting individual variables as well as its possible interactions in SPSS, and sequentially eliminated from the model, the least important variable. The importance of a variable was judged by the size of the t (or equivalent F)-statistic for dropping the variable from the model, i.e., the t -statistic was used for testing whether the corresponding regression coefficient is 0. Initially, the independent variable with the largest p -value, as shown in the coefficient table, would be omitted the regression analysis was rerun on the remaining variables. The insignificant variables were omitted from the model by eliminating any independent variables with the largest p -value and greater than 0.05. The backward elimination process would then be repeated and the model after subsequent iteration would be denoted by *Ma.b.c* where 'c' is the respective number of insignificant variables eliminated. The number of runs or iterations would end when all the significant variables had p -values less than 0.05 (Noraini *et al.* (2008); Zainodin *et al.* (2011)).

c) Coefficient Tests

Considering a general model of equation (2), all possible models would undergo the Global and Coefficient tests. The opposing hypotheses of the Global test were:

$$H_0 : \Omega_1 = \Omega_2 = \dots = \Omega_k = 0, \text{ and } H_1 : \text{at least one of the } \Omega \text{'s is non-zero.}$$

From the ANOVA table of the selected model, missing independent variables could be known from the degree of freedom and the number of excluded variables due to multicollinearity. Null hypothesis, H_0 would be rejected if $F_{cal} > F^*_{(k, n-k-1, \alpha)}$ and vice versa. Alternatively, if the p -value in the ANOVA table was less than α , null hypothesis would also be rejected, implying that at least an independent variable would have an effect on the dependent variable.

The Coefficient test would determine the significance of the corresponding independent variables on the model. The opposing hypotheses of the j -th coefficient test were:

$$H_0 : \Omega_j = 0, \text{ and } H_1 : \Omega_j \neq 0$$

where Ω_j is the coefficient of W_j for $j=1, 2, 3, \dots, k$.

The Wald test was then carried out to ensure that the removed or eliminated variables were positively identified, besides testing the joint significance of several regression coefficients of the independent variables. By assuming, model before omission of independent variable as *Unrestricted model (U)*, and model after one or more independent variable(s) being eliminated as *Restricted model (R)*, opposing hypotheses were used to test the overall significance for both the restricted and the unrestricted models.

$$(U) V = \Omega_0 + \Omega_1 X_1 + \Omega_2 X_2 + \dots + \Omega_m X_m + \Omega_{m+1} X_{m+1} + \dots + \Omega_k X_k + u$$

$$(R) V = \Omega_0 + \Omega_1 W_1 + \Omega_2 W_2 + \dots + \Omega_m X_m + v$$

The opposing hypotheses of the Wald test were:

$$H_0 : \Omega_{m+1} = \Omega_{m+2} = \dots = \Omega_k = 0,$$

and

$$H_1 : \text{at least one of } \Omega_j \text{'s is not zero.}$$

The critical value was denoted by $F_{(k-m, n-k-1, \alpha)}$, found in the F-distribution table at α percent level of significance. Using the F-statistics (Christensen (1996)), when $F_{cal} < F_{(k-m, n-k-1, \alpha)}$, the null hypothesis was not rejected, hence accepting the removal of the omitted variables. Thus, there would be no significant contribution on the dependent variable at α percent level of significance.

8. EIGHT CRITERIA MODEL SELECTION

In recent years, several criteria for choosing among models have been proposed. These entire selection criteria take the form of the residual sum of squares (*SSE*) multiplied by a penalty factor that depends on the complexity of the model. A more complex model will reduce *SSE* but raise the penalty. The criteria thus provide other types of trade-offs between goodness of fit and model complexity. A model with a lower value of a criterion statistics is judged to be preferable (Christensen (1996)).

Ramanathan (2002) had also shown the statistical procedures of getting the best model based on these eight selection criteria, namely, *SGMASQ*, *AIC*, *FPE*, *GCV*, *HQ*, *RICE*, *SCHWARZ* and *SHIBATA* as shown by the Table 3 below. The best model which could give the volume would then be chosen based on these eight selection criteria. The eight selection criteria (*8SC*) for the general model is based on $K = (k + 1)$ estimated parameters, n is the number of observations and *SSE* is the sum square error.

TABLE 3: Eight Selection Criteria on General Model

<i>AIC</i> (Akaike (1970))	$\left(\frac{SSE}{n}\right)e^{(2K/n)}$	<i>RICE</i> (Rice (1984))	$\left(\frac{SSE}{n}\right)\left[1 - \left(\frac{2K}{n}\right)\right]^{-1}$
<i>FPE</i> (Akaike (1974))	$\left(\frac{SSE}{n}\right)\frac{n+K}{n-K}$	<i>SCHWARZ</i> (Schwarz (1978))	$\left(\frac{SSE}{n}\right)n^{K/n}$
<i>GCV</i> (Golub <i>et al.</i> (1979))	$\left(\frac{SSE}{n}\right)\left[1 - \left(\frac{K}{n}\right)\right]^{-2}$	<i>SGMASQ</i> (Ramanathan (2002))	$\left(\frac{SSE}{n}\right)\left[1 - \left(\frac{K}{n}\right)\right]^{-1}$
<i>HQ</i> (Hannan and Quinn (1979))	$\left(\frac{SSE}{n}\right)(\ln n)^{2K/n}$	<i>SHIBATA</i> (Shibata (1981))	$\left(\frac{SSE}{n}\right)\frac{n+2K}{n}$

9. RESIDUAL ANALYSES ON BEST MODEL

Best model was identified based on the eight selection criteria (8SC), and carried out after the regression analyses and hypotheses testing mentioned above. The goodness-of-fit, being one of the attributes of the best model, would demonstrate the variations of the dependent variable and the distribution of the error terms. Randomness test and normality test were conducted to examine these attributes (Gujarati (2006); Ismail *et al.* (2007)). Using the best model to obtain the estimated values, the residual or error term, u which was the difference between the actual and the estimated values of the best model could thus be calculated. Since there would be a pattern between the error terms and the n th observations, the error terms implied homoscedasticity (Gujarati (2006)).

The randomness test of the residuals was carried out to test the accuracy of the best model. The opposing hypotheses were $H_0 : mean(u_i) = E(u) = 0$ and $H_1 : mean(u_i) = E(u) \neq 0$. The t -

statistics ($|T_{cal}|$) was calculated using the formula $T_{cal} = R \sqrt{\frac{n-k-1}{1-R^2}}$

$$\text{where } R = \frac{\frac{1}{n} \sum_{i=1}^n i u_i - \bar{u} \bar{K}}{S_u S_1}, \quad \bar{u} = \frac{1}{n} \sum_{i=1}^n u_i, \quad S_u^2 = \frac{1}{n} \sum_{i=1}^n (u_i - \bar{u})^2, \quad S_1 = \frac{n^2 - 1}{12}$$

and $\bar{K} = \frac{n+1}{2}$. With n observations and k estimated parameters, using the

normal distribution table, accept null hypothesis if $|T_{cal}| < z_{\alpha/2}^*$ at five percent level of significance. $|T_{cal}|$ was to be calculated, while $z_{0.025}^*$ at five percent level of significance was 1.96. Acceptance of the null hypothesis implies that the mean error would be zero and the error terms are randomly distributed and independent of one another.

10. ANALYSES OF MULTIPLE REGRESSION (MR) MODELS

Descriptive Statistics, Correlation Matrix and Multicollinearity

The data sets were tested for normality. Since they were not normally distributed, transformations of the variables were then carried out using Kolmogorov-Smirnov statistics at Lilliefors significance level of more than 0.05. Before transformation, only two variables, namely TH and Dt , were normal, having their significant p -values of more than 0.05. Appropriate

ladder-power transformations were then used for normality. Table 4 indicates the newly assigned variables for the transformed variables.

TABLE 4: Definition of New Variables

Transformed	New Variable	Definition
$VH^{0.835}$	V1	Volume of stem (m^3): Huber's
$VN^{0.9}$	V2	Volume of stem (m^3): Newton's
h^3	X1	Stem height, h (m)
TH	X2	Tree height from ground to the peak (m)
$Dbh^{2.7}$	X3	Diameter at breast height
Dm^2	X4	Diameter at middle of stem
Dt	X5	Diameter at top of stem

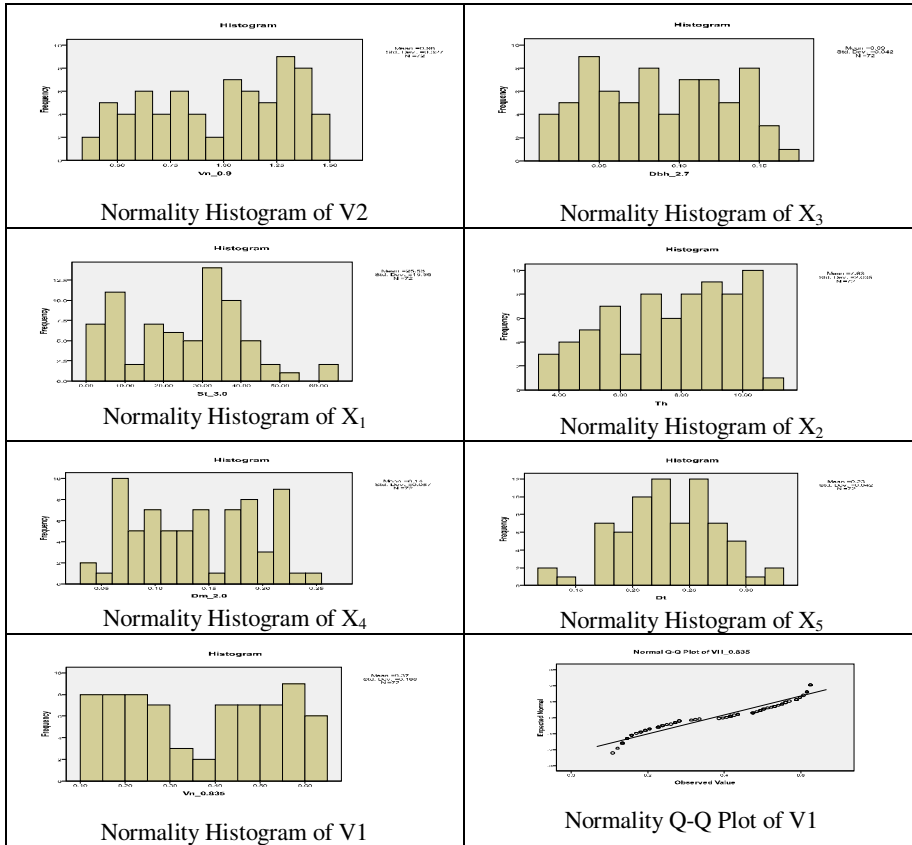
All the new variables, except for $(V1=VH^{0.835})$, had turned to normal, as shown in Table 5 with their p -values greater than 0.05 (in bold). The best power of ladder transformation for VH was 0.835. Based on the p -value of the *Kolmogorov-Smirnov (K-S)* statistics, normality was not assumed, as shown in Table 5.

TABLE 5: Descriptive Statistics of Normality Tests of Variables after Transformation

Definition of New Variables	After Transformation						
	V1	V2	X1	X2	X3	X4	X5
Mean	0.3726	0.978	25.53	7.63	22104	1381	23.15
Standard Error	0.0198	0.386	1.81	0.24	1233.4	66.920	0.499
Std. Deviation	0.1683	0.327	15.36	2.036	10466	567.8	4.237
Minimum	0.11	0.38	1.48	3.39	4476.0	412.9	12.0
Maximum	0.63	1.46	61.63	11.01	41213	2500	33.0
Skewness	-0.020	-0.23	0.182	-0.36	0.007	0.037	-0.14
Kurtosis	-1.494	-1.31	-0.72	-0.86	-1.27	-1.26	0.13
<i>K-S</i> Statistics	0.134	0.104	0.099	0.103	0.104	0.104	0.055
<i>K-S</i> (p -value)	0.003	0.050	0.077	0.055	0.052	0.051	0.200

Graphically, the assumptions of normality were supported by the normality histogram plots of the new variables as depicted in Table 6 below. The Q-Q plot of V1 had further supported the acceptance of the relatively normal histogram plot for V1, since all the points were along the straight line without deviation and no presence of outliers.

TABLE 6: Normality Plots of the New Variables.



The data sets were initially tested for bivariate relationships between the main variables using the Pearson Correlation Coefficient test. From the Correlation Coefficient matrix, there existed positive relationships from weak (a value of $|r| = 0.226$) to strong (a value of $|r| = 0.952$) between the variables, significant at the 0.01 level (2-tailed). Table 7 shows the correlation coefficients with multicollinearity using the Huber's formula for *R. regia*.

TABLE 7: Correlation Coefficient Matrix with Multicollinearity using Huber's Formula.

Transformed Variables	V1	X1	X2	X3	X4	X5
V1	1					
X1	0.646	1				
X2	0.706	0.842	1			
X3	0.906	0.380	0.468	1		
X4	0.922	0.324	0.463	0.952	1	
X5	0.722	0.226	0.373	0.809	0.788	1

There was a strong linear relationship between X3 and X4 of the main variables giving a value of $|r| = 0.952$. This was expected in the data sets since the diameter at breast height was technically measured 1.3 meters from the base of tree trunk. From observations of the trees, the middle of tree stem occasionally fell within the range of the diameter at breast height, *Dbh*. However, the existence of multicollinearity ($|r| > 0.95$ (in bold)) between the variables had to be remedied first so as to overcome the presence of any excluded variables when undergoing the elimination processes.

The multicollinearity effect was thus eliminated by first investigating the effect of variables X3 and X4 on the volume, say for model M31 using the variable V1 (volume using the Huber's formula) with five single independent variables without interactions. As shown in Table 7, this was a multicollinearity of Case C type which had a single tie of a high correlation coefficient value amongst these variables ($|r| = 0.952 > 0.95$). More details on the types of multicollinearity cases and the remedial techniques in removing multicollinearity can be found in Zainodin *et al.* (2011).

Variable X3 having the lower absolute correlation coefficient on the volume ($|r| = 0.906$), compared to X4 ($|r| = 0.922$), was thus eliminated. Model M31H had then reduced to M31.1H. The value 1 denoted the letter 'b', the first eliminated source variable of multicollinearity. Rerunning the model after elimination, the coefficient matrix would thus show the nonexistence of multicollinearity, as shown by Table 8 below. The equation of the model would thus have variables without high multicollinearity. The model could then undergo the next process of Phase 2, that is, the backward elimination of insignificant variables.

TABLE 8: Correlation Coefficient Matrix without Multicollinearity using Huber's Formula.

Transformed Variables	V1	X1	X2	X4	X5
V1	1				
X1	0.646	1			
X2	0.706	0.842	1		
X4	0.922	0.324	0.463	1	
X5	0.722	0.226	0.373	0.788	1

Similar procedures using the Zainodin-Noraini Multicollinearity Remedial Techniques (Zainodin *et al.* (2011)) were carried out on all the regression models using the Newton's formula. It could also be seen from Table 9 that there was a strong correlation between X3 and X4.

TABLE 9: Correlation Coefficient Matrix with Multicollinearity using Newton's Formula.

Transformed Variables	V2	X1	X2	X3	X4	X5
V2	1					
X1	0.811	1				
X2	0.814	0.842	1			
X3	0.806	0.380	0.468	1		
X4	0.797	0.324	0.463	0.952	1	
X5	0.634	0.226	0.373	0.809	0.788	1

Similarly as before, the existence of multicollinearity ($|r| > 0.95$ (in bold)) between the variables had to be remedied first by eliminating the source variable of multicollinearity. Variable X4 having the lower absolute correlation coefficient on the volume ($|r| = 0.797$) compared to X3 ($|r| = 0.806$), was thus eliminated. Model M31N had then reduced to M31.1N with the value 1, denoting the first eliminated source variable of multicollinearity. The correlation matrix was thus without multicollinearity, and is shown in Table 10.

TABLE 10: Correlation Coefficient Matrix without Multicollinearity using Newton's Formula

Transformed Variables	V2	X1	X2	X3	X5
V2	1				
X1	0.811	1			
X2	0.814	0.842	1		
X3	0.806	0.380	0.468	1	
X5	0.634	0.226	0.373	0.809	1

Multicollinearity also occurred in models where there existed a high frequency of a common variable of a high correlation with the other independent variables, as shown by model M39.0.0 in Table 11. Variable X3 with the highest frequency of 2 and a lower absolute correlation value with the dependent variable, would have to be eliminated first.

In addition, for models with variables having the highest correlation with other independent variables, would also be eliminated due to multicollinearity, such as, variables X13 and X14 which had a high correlation coefficient of 0.986. Interaction variable X13 would therefore be removed since it had a lower correlation value with the volume. The model was then rerun.

TABLE 11: Model with High Frequency Multicollinearity.

M39.0.0	V1	X1	X3	X4	X13	X14	X34
V1	1						
X1	0.646	1					
X3	0.906	0.380	1				
X4	0.922	0.324	0.952	1			
X13	0.903	0.826	0.781	0.708	1		
X14	0.925	0.857	0.765	0.728	0.986	1	
X34	0.913	0.347	0.970	0.971	0.744	0.744	1

A single tie of a high correlation value of 0.971 indicated that the interaction variable X34 had to be removed due to its lower impact on the volume. Rerunning the correlation again would result in Table 12 where model M39.3.0 had gone through three multicollinearity source removals of the Zainodin-Noraini multicollinearity remedial techniques (Zainodin *et al.* (2011)).

TABLE 12: Final Correlation Matrix without High Multicollinearity

M39.3.0	V1	X1	X4	X14
V1	1			
X1	0.646	1		
X4	0.922	0.324	1	
X14	0.925	0.857	0.728	1

Parameter Tests of Model Functions

All possible models would undergo the procedures of multicollinearity reduction, as mentioned in the earlier section. Parameter tests corresponding to the independent variables are therefore carried out to verify the insignificance of the excluded variables. The parameter tests (excluding constant of the model) would include the Global test, Coefficient test and the Wald test. The rejection of the Global test null hypothesis would imply that at least one variable would have an effect on the relative dependent variable as implicated by model M71.6 which had undergone six removals of multicollinearity source variables. From the ANOVA table in Table 13, since $F_{cal} = 4615.36 > F^*(7, 64, 0.05) = 2.164$, the null hypothesis would be rejected implying that at least one of the parameters was not zero. The p -value in the ANOVA table also showed that it was less than 0.05, at five percent level of significance, thus rejecting the null hypothesis.

TABLE 13: ANOVA Table for Model M71.6.0H

Model	Source	Sum of Squares	df	Mean Square	F	Sig. (p-value)
M71.6.0H - Huber's	Regression	2.0114	7	0.2873	4615.36	5.9x10 ⁻⁸⁴
	Residual	0.0039	64	6.2x10 ⁻⁵		
	Total	2.0154	71			

The models for each volumetric formula of *R.regia* were then selected by applying the backward elimination method of the Coefficient test (Peck *et al.* (2008)). This Coefficient test was done to determine the significance of the related independent variable. An illustration of the elimination procedure is shown in the following table using Huber's formula.

TABLE 14: Coefficient Table of Model M71.6.0H

Model M71.6.0H – Huber's	Unstandardized Coefficients		t	Sig. p-value
	B	Std.Error		
(Constant)	-8.772x10 ⁻²	1.777x10 ⁻²	-4.936	5.993x10 ⁻⁶
X1	4.123x10 ⁻⁴	2.7x10 ⁻⁴	1.504	0.1376
X5	1.117x10 ⁻²	1.611x10 ⁻³	6.934	2.431x10 ⁻⁹
X13	1.164x10 ⁻³	2.193x10 ⁻⁴	5.307	1.488x10 ⁻³
X15	-1.899x10 ⁻³	5.766x10 ⁻⁴	-3.299	1.585x10 ⁻³
X34	2.044x10 ⁻⁴	1.408x10 ⁻⁵	14.518	6.361x10 ⁻²²
X35	-4.186x10 ⁻⁴	2.837x10 ⁻⁵	-14.758	2.826x10 ⁻²²
X145	1.098x10 ⁻⁴	8.878x10 ⁻⁶	12.369	1.261x10 ⁻¹⁸

From Table 14, based on the parameter regression coefficient of variable X1 with p -value=0.1376, above the value of 0.05, and smallest absolute t -statistics or the highest p -value, as compared to the rest of the models, therefore, variable X1 was removed and the model was then refitted. The remaining variables were then rerun and any variable which had the new highest p -value (>5%) was subsequently removed. The elimination procedures had been described in detail by Noraini *et al.* (2008). Table 15 below shows the final coefficients of the variables of the possible best model (M71.6.1H) after the first regression procedure. There were no p -values more than 0.05 (>5%), hence, the selected model M71H had reached to its final stage of the selected best model.

TABLE 15: Coefficient Table of Model M71.6.1H after 1st Iteration.

Model M71.6.1H – Huber’s	Unstandardized Coefficients		t	Sig. p-value
	B	Std.Error		
(Constant)	-8.853x10 ⁻²	1.793x10 ⁻²	-4.936	5.847x10 ⁻⁶
X5	1.077x10 ⁻²	1.604x10 ⁻³	11.292	5.521x10 ⁻⁹
X13	1.313x10 ⁻³	1.972x10 ⁻⁴	24.892	6.886x10 ⁻⁹
X15	-1.656x10 ⁻³	5.578x10 ⁻⁴	-9.934	4.177x10 ⁻³
X34	2.010x10 ⁻⁴	1.403x10 ⁻⁵	9.658	8.729x10 ⁻²²
X35	4.182x10 ⁻⁴	2.863x10 ⁻⁵	9.250	3.369x10 ⁻²²
X145	-1.067x10 ⁻⁴	8.729x10 ⁻⁶	-7.984	1.615x10 ⁻¹⁸

The Wald test was then carried out to justify the elimination of the insignificant independent variables from the selected best models. Using the Huber’s formula, the unrestricted model was given as (U_H) while the restricted model was (R_H).

$$\begin{aligned}
 &U_H (M71.6.0): \\
 &V1 = \beta_0 + \beta_1 X_1 + \beta_5 X_5 + \beta_{13} X_{13} + \beta_{15} X_{15} \\
 &\quad + \beta_{34} X_{34} + \beta_{35} X_{35} + \beta_{145} X_{145} + u
 \end{aligned} \tag{4}$$

$$\begin{aligned}
 &R_H (M71.6.1): \\
 &V1 = \beta_0 + \beta_5 X_5 + \beta_{13} X_{13} + \beta_{15} X_{15} \\
 &\quad + \beta_{34} X_{34} + \beta_{35} X_{35} + \beta_{145} X_{145} + v
 \end{aligned} \tag{5}$$

where u and v are error terms. Using the opposing hypotheses and ANOVA table for both the unrestricted and restricted models, the null hypothesis would be accepted when $F_{cal} < F_{(k-m, n-k-1, \alpha)}$, implying that the eliminated independent variable would have insignificant effect or contribution on the

relative dependent variable. The Wald test for model M71 was shown by the ANOVA Table in Table 16 below.

Since $F_{cal} = 3.2165 < F(1, 64, 0.05) = 3.9983$, the null hypothesis is accepted at five percent level of significance. The removal of the insignificant variables is acceptable since they have no contribution to the dependent variable, i.e. on the volume.

TABLE 16: ANOVA Table for Wald Test of Model M71.6.0H

Model	Source	Sum of Squares	df	Mean Square	F
M71.6.0 H- Huber's	<i>Difference (R-U)</i>	2.0×10^{-4}	1	2.0×10^{-4}	3.2165
	<i>Unrestricted (U)</i>	3.98×10^{-3}	64	6.218×10^{-5}	
	<i>Restricted (R)</i>	4.13×10^{-3}	65		

These modelling procedures were sequentially repeated for the other possible models using the Newton's formula. Starting with the removal of multicollinearity source variables, validation of the excluded variables using the Global test, elimination of the insignificant variables using the Coefficient test (determining that there would be no more variable with a p -value of more than 0.05), and finally, the Wald test to positively justified the elimination of the insignificant variables from the model. Similarly, the Wald test using the Newton's formula, the unrestricted model was given by (U_N) while the restricted model was (R_N).

$$U_N (M77.7.0):$$

$$V2 = \beta_0 + \beta_2 X_2 + \beta_3 X_3 + \beta_4 X_4 + \beta_5 X_5 + \beta_{12} X_{12} + \beta_{15} X_{15} + \beta_{23} X_{23} + \beta_{25} X_{25} + u \quad (6)$$

$$R_N (M77.7.1):$$

$$V2 = \beta_0 + \beta_2 X_2 + \beta_4 X_4 + \beta_5 X_5 + \beta_{12} X_{12} + \beta_{15} X_{15} + \beta_{23} X_{23} + \beta_{25} X_{25} + v \quad (7)$$

TABLE 17: Coefficient Table of Model M77.7.1N after 1st Iteration.

Model M77.7.1N - Newton's	Unstandardized Coefficients		<i>t</i>	Sig. <i>p</i> -value
	B	Std.Error		
(Constant)	-0.1813	0.0801		2.7x10 ⁻²
X2	0.1190	0.0135	0.7396	1.1689x10 ⁻¹²
X4	2.5239	0.2159	0.4378	1.5862x10 ⁻¹⁷
X5	1.0206	0.3655	0.1321	6.88x10 ⁻³
X12	-1.2x10 ⁻³	1.4709x10 ⁻⁴	-0.5691	5.1063x10 ⁻¹¹
X15	9.3x10 ⁻⁴	5.5361x10 ⁻⁵	1.0909	3.7873x10 ⁻²⁵
X23	3.6154x10 ⁻⁷	1.4634x10 ⁻⁷	0.1199	3.14x10 ⁻²
X25	-3.9173x10 ⁻³	5.8254x10 ⁻⁴	-0.7762	5.6511x10 ⁻⁹

11. NEWTON'S-HUBER'S MULTIPLE REGRESSION (MR) MODELS

After undergoing the procedures in Phase 1 and Phase 2, the number of possible models had reduced to 34 selected models using the Huber's formula and 42 selected models using the Newton's formula. Based on the regression models, without multicollinearity and insignificant variables, the final selected regression model functions for *R.regia*, using the Huber's formula, were then tabulated based on the eight selection criteria, as mentioned earlier. The best model was chosen when it had majority of the least value of the eight criteria, as indicated by M79.21.0H in Table 18 below. The model equation is:

M79.21.0H :

$$\begin{aligned}
 V1 = & -0.110 - 1.16x10^{-3} X_1 + 1.01x10^{-2} X_5 + 2.16x10^{-3} X_{12} - 3.49x10^{-3} X_{14} \\
 & - 2.11x10^{-3} X_{15} - 1.74x10^{-4} X_{24} + 1.0x10^{-4} X_{34} \\
 & - 2.3x10^{-4} X_{35} + 1.1x10^{-6} X_{1345} + v
 \end{aligned}
 \tag{8}$$

TABLE 18: Selected Best Models on 8SC Using Huber's Formula.

Selected model	k+1	SSE	AIC	FPE	GCV	HQ	RICE	SCHWARZ	SGMASQ	SHIBATA
M1.0.0	2	1.17328	0.01723	0.01723	0.02856	0.01506	0.01725	0.01835	0.01833	0.01718
M2.0.0	2	1.01297	0.01487	0.01487	0.02856	0.01506	0.01489	0.01584	0.01447	0.01720
M3.0.0	2	0.36213	0.00531	0.00532	0.02856	0.01506	0.00533	0.00566	0.00517	0.00531
M4.0.0	2	0.30496	0.00448	0.00448	0.02856	0.01506	0.00448	0.00448	0.00436	0.00447
:	:	:	:	:	:	:	:	:	:	:
M11.0.0	3	0.10618	0.00160	0.00160	0.04408	0.03135	0.00161	0.00176	0.00154	0.00160
M12.0.0	3	0.51816	0.00528	0.00528	0.04408	0.03135	0.00529	0.00580	0.00507	0.00526
:	:	:	:	:	:	:	:	:	:	:
M33	3	0.08211	0.00124	0.00124	0.04408	0.03135	0.00124	0.00136	0.00119	0.00124
M34	3	0.00598	0.00009	0.00009	0.06049	0.04897	0.00009	0.00001	0.00008	0.00009
M39	2	0.28864	0.00424	0.00424	0.02856	0.01505	0.00424	0.00451	0.00412	0.00423
:	:	:	:	:	:	:	:	:	:	:

TABLE 18 (continued): Selected Best Models on 8SC Using Huber's Formula.

Selected model	k+1	SSE	AIC	FPE	GCV	HQ	RICE	SCHWARZ	SGMASQ	SHIBATA
M58.2.1	5	0.07296	0.00116	0.00116	0.07785	0.06797	0.00118	0.00136	0.00109	0.00115
:	:	:	:	:	:	:	:	:	:	:
M63.2.1	5	0.01208	0.00019	0.00019	0.07785	0.06795	0.00019	0.00023	0.00018	0.00019
M64.4.1	3	0.10073	0.00152	0.00152	0.04408	0.03135	0.00153	0.00167	0.00146	0.00151
M65.3.1	4	0.16401	0.00255	0.00255	0.06049	0.04897	0.00256	0.00289	0.00241	0.00253
M68.8.2	4	0.01828	0.00028	0.00028	0.06049	0.04897	0.00029	0.00032	0.00269	0.00028
M70.6.0	8	0.01022	0.00018	0.00018	0.13631	0.13427	0.00018	0.00023	0.00016	0.00017
M71.6.1	7	0.00413	0.00006	0.00006	0.11570	0.11054	0.00007	0.00008	0.00006	0.00007
M72.8.2	5	0.09161	0.00146	0.00146	0.07785	0.06797	0.00148	0.00136	0.00171	0.00145
M79.21.0	10	0.00183	0.00003	0.00003	0.18140	0.18716	0.00003	0.00004	0.00003	0.00003
Min		0.00183	0.00003	0.00003	0.02856	0.01505	0.00003	0.00004	0.00003	0.00003

Similarly, the best MR model using the Newton's formula is given by M80.22.ON as in Table 19 below:

$$\begin{aligned}
 M\ 80.22.ON : \\
 V^2 = & -0.184 + 0.125 X_2 + 1.734 X_4 + 1.270 X_5 \\
 & -2.308 \times 10^{-3} X_{12} + 1.419 \times 10^{-3} X_{15} \\
 & +1.084 \times 10^{-6} X_{23} - 4.637 \times 10^{-3} X_{25} \\
 & +5.564 \times 10^{-7} X_{124} - 1.402 \times 10^{-8} X_{135} + v
 \end{aligned} \tag{9}$$

TABLE 19: Selection of Best Models on 8SC using Newton's Formula.

Selected model	k+1	SSE	AIC	FPE	GCV	HQ	RICE	SCHWARZ	SGMASQ	SHIBATA
M1	:	2.6039	0.03720	0.03823	0.03823	0.03826	0.03921	0.03829	0.04073	0.03817
:	:	:	:	:	:	:	:	:	:	:
M56.1.0	4	0.3440	0.00506	0.00534	0.00534	0.00535	0.00561	0.00538	0.00606	0.00530
:	:	:	:	:	:	:	:	:	:	:
M60.1.1	3	0.8087	0.01171	0.01219	0.01219	0.01221	0.01266	0.01224	0.01341	0.01215
:	:	:	:	:	:	:	:	:	:	:
M76.1.2	3	0.0311	0.00499	0.00520	0.00519	0.00520	0.00539	0.00521	0.00571	0.00518
M77.7.1	7	0.0653	0.00103	0.00113	0.00113	0.00115	0.00125	0.00117	0.00146	0.00111
M78.18.1	7	0.0114	0.00282	0.00313	0.00314	0.00318	0.00346	0.00323	0.00403	0.00307
M79.21.1	8	0.1932	0.00301	0.00334	0.00334	0.00338	0.00369	0.00343	0.00430	0.00327
M80.22.0	9	0.0333	0.00053	0.00059	0.00060	0.00060	0.00067	0.00062	0.00079	0.00058
Min		0.0333	0.00053	0.00059	0.00060	0.00060	0.00067	0.00062	0.00079	0.00058

However, comparing the minimum values of the eight selection criteria (8SC) of the best models from Table 18 and Table 19 respectively, the regression model (M79.21.OH) using the Huber's formula is better with the least SSE (1.83×10^{-3}). By transforming the defined variables back into the model equation, the best MR model using the Huber's formula is therefore

M79.21.0 H :

$$\begin{aligned}
 VH^{0.835} = & -0.110 - 1.16 \times 10^{-3} h^3 + 1.01 \times 10^{-2} Dt + 2.16 \times 10^{-3} h^3 .TH - \\
 & 3.49 \times 10^{-3} h^3 .Dm^2 - 2.11 \times 10^{-3} h^3 .Dt - 1.74 \times 10^{-4} TH .Dm^2 + \\
 & 1.0 \times 10^{-4} Dbh^{2.7} .Dm^2 - 2.3 \times 10^{-4} Dbh^{2.7} .Dt + \\
 & 1.1 \times 10^{-6} h^3 Dbh^{2.7} .Dm^2 .Dt + v
 \end{aligned}
 \tag{10}$$

12. RESIDUAL ANALYSES

Using the best model to obtain the estimated values, the residual or error term, ε , which is the difference between the actual and the estimated values of the best model was then calculated. Since there was no obvious pattern between the error terms and the n observations, hence, the error terms implied homoscedasticity (Ismail *et al.* (2007)).

The randomness test of the residuals was also carried out to test the accuracy of the best model (M79.21.0H). The opposing hypotheses were: $H_0 : mean(u_i) = E(u) = 0$ and $H_1 : mean(u_i) = E(u) \neq 0$. Since the data set contained 72 observations, using the normal distribution table, accept null hypothesis if $|T_{cal}| < z_{\alpha/2}^*$ at five percent level of significance. The t -statistics ($|T_{cal}|$) was calculated at 0.15433 while $z_{0.025}^*$ at five percent level of significance was 1.96. Hence, the null hypothesis is accepted, implying that the mean error is zero and the terms are randomly distributed and independent of one another as shown by the residual plots in Figure 3.

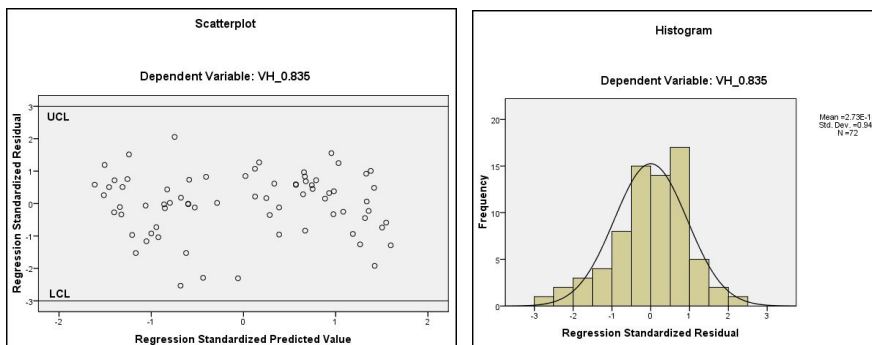


Figure 3: Scatter Plot and Histogram of the Standardized Residuals.

Normality test of the residuals was then conducted to examine whether the error terms are normally distributed using Kolmogorov-Smirnov statistics (for $n > 50$).

TABLE 20: Residuals Normality Test for Model M79.21.0H

Residuals of M79.21.0H	Kolmogorov-Smirnov		
	Statistic	d.f.	Sig.
	0.117	71	0.052

The distribution is considered normal if the significant p -value was greater than 0.05 at five percent level of significance as shown in Table 20 above. Thus, this implies that the error terms were normally distributed and all other data diagnostics were also satisfactory. From the plots of the randomness and normality tests of the residuals, it can be concluded that assumptions of homoscedasticity, randomness and normality of the best model have been satisfied.

13. DISCUSSIONS

Stem volume is a good estimation in determining the biomass of a certain tree species where various estimation techniques can be employed. In this paper, models were developed using multiple regression techniques based on the Newton's and Huber's formulae. At the preliminary stage, multicollinearity remedial methods were employed to reduce biased estimators. Comparisons were then made based on the eight selection criteria for the best model.

From the results obtained, it is clear that both volumetric equations using Huber's and Newton's formulae for *R. regia* can be estimated using multiple regression models. Using Huber's volumetric formula for *R. regia*, the best model is found to be as in equation (8), with two main and seven interaction variables. Meanwhile, under Newton's volumetric formula the best model is found to be as in equation (9) with three main and six interaction variables. Based on the 8SC, the Huber MR model is the best regression model since it gives the least SSE value, and satisfies majority of the requirements of the criteria.

Under these equations, it is also obvious that stem height or bole (h) and diameter at the top (Dt) for *R. regia* are major contributors towards the stem volume and significant contributions are also from the tree height (TH)

and mensuration diameters at breast height (Dbh) and the middle, Dm . It should be noted that there is a common directly positive linear relationship from the diameter at the top (Dt) in both models. The difference existed in the Huber's and Newton's models are due to the volumetric formulae themselves. Newton's formula takes into account the areas at the base (Dbh), middle, and the top while the Huber's formula accounted just a single diameter halfway along the stem. This had also been concluded by Noraini *et al.* (2008) on a tropical tree species, *C.iners* where the Newton's regression model gave a better estimation with respect to the mensuration data obtained. Onyekwelu (2004) had also concluded that most of the biomass accumulation was stored in the stem. This indicated that a very high percentage of tree wood could be merchantable either for timber or other uses. Fuwape *et al.* (2001) also showed that more than 75% of total biomass yield were from the stem too, for both species of *Gmelina arborea* and *Nauclea diderrichii* stands in the Akure forest reserve where the research was done.

14. CONCLUSIONS

Removal of multicollinearity and elimination of insignificant variables have primarily reduced the backward elimination procedures and the number of selected models. Consequently, the iteration time and selection of best models are also reduced. The process of eight selection criteria (8SC) is a convenient way to determine or select the best model for stem volume estimation. The volumetric stem biomass of the *R.regia* trees of the paraboloid shape is best modeled by adopting the Huber's volumetric formula. Contrary to the *C.iners* of Noraini *et al.* (2008) which is the trapezoidal shape, the Huber's volumetric equation is significantly preferable due to its simplicity in estimating the stem biomass of the paraboloid shaped trees of *R. regia*.

The best models using both the Huber's and Newton's formulae can also be used to estimate missing data values for tree volume prediction. In forecasting, such as time series data where discrete variables involving time and space are measurable items, any missing values will be a deterrent to model formulation. Models developed for forecasting with a mean average prediction error (MAPE) of less than 10% would give very good estimates, and hence, lower inventory costs in tree-planting and forest management practices as well as felling or logging costs in timber production.

Both the two best models can be used for estimation, however, the Huber's best model will be able to give good estimates with two single variables (stem height and diameter at the top) and variables up to the first order interactions can be represented in the model. The third order interaction is of minimal effect towards the volume. Meanwhile, the Newton's best model has many single independent variables (tree height, diameter at the middle, and diameter at the top) and interaction variables up to the second order to affect the volume estimation.

Further research can be done towards the numerical analysis of stem biomass by looking into other volumetric equations. Since maximum volumetric biomass can also be related to the circumferential area of tree trunk, optimization of merchantable tree log and its economic values (Lemmens *et al.* (1995)) can also be explored further with forecasting into the potentials and commercialization of tree species. It is suggested that stem biomass estimation with respect to shape, the techniques involved and the numerical problems being addressed, will certainly give a new outlook in estimating and modeling of the volumetric stem biomass.

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Appendix A: All Possible Regression Models

Models with Five Single Independent Variables	
M1	$Y_1 = \beta_0 + \beta_1 X_1 + u$
M2	$Y_2 = \beta_0 + \beta_2 X_2 + u$
M3	$Y_3 = \beta_0 + \beta_3 X_3 + u$
M4	$Y_4 = \beta_0 + \beta_4 X_4 + u$
M5	$Y_5 = \beta_0 + \beta_5 X_5 + u$
M6	$Y_6 = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + u$
M7	$Y_7 = \beta_0 + \beta_1 X_1 + \beta_3 X_3 + u$
M8	$Y_8 = \beta_0 + \beta_1 X_1 + \beta_4 X_4 + u$
M9	$Y_9 = \beta_0 + \beta_1 X_1 + \beta_5 X_5 + u$
M10	$Y_{10} = \beta_0 + \beta_2 X_2 + \beta_3 X_3 + u$
M11	$Y_{11} = \beta_0 + \beta_2 X_2 + \beta_4 X_4 + u$
M12	$Y_{12} = \beta_0 + \beta_2 X_2 + \beta_5 X_5 + u$
M13	$Y_{13} = \beta_0 + \beta_3 X_3 + \beta_4 X_4 + u$
M14	$Y_{14} = \beta_0 + \beta_3 X_3 + \beta_5 X_5 + u$
M15	$Y_{15} = \beta_0 + \beta_4 X_4 + \beta_5 X_5 + u$

M16	$Y_{16} = \beta_0 + \beta_1X_1 + \beta_2X_2 + \beta_3X_3 + u$
M17	$Y_{17} = \beta_0 + \beta_1X_1 + \beta_2X_2 + \beta_4X_4 + u$
M18	$Y_{18} = \beta_0 + \beta_1X_1 + \beta_2X_2 + \beta_5X_5 + u$
M19	$Y_{19} = \beta_0 + \beta_1X_1 + \beta_3X_3 + \beta_4X_4 + u$
M20	$Y_{20} = \beta_0 + \beta_1X_1 + \beta_3X_3 + \beta_5X_5 + u$
M21	$Y_{21} = \beta_0 + \beta_1X_1 + \beta_4X_4 + \beta_5X_5 + u$
M22	$Y_{22} = \beta_0 + \beta_2X_2 + \beta_3X_3 + \beta_4X_4 + u$
M23	$Y_{23} = \beta_0 + \beta_2X_2 + \beta_3X_3 + \beta_5X_5 + u$
M24	$Y_{24} = \beta_0 + \beta_2X_2 + \beta_4X_4 + \beta_5X_5 + u$
M25	$Y_{25} = \beta_0 + \beta_3X_3 + \beta_4X_4 + \beta_5X_5 + u$
M26	$Y_{26} = \beta_0 + \beta_1X_1 + \beta_2X_2 + \beta_3X_3 + \beta_4X_4 + u$
M27	$Y_{27} = \beta_0 + \beta_1X_1 + \beta_2X_2 + \beta_3X_3 + \beta_5X_5 + u$
M28	$Y_{28} = \beta_0 + \beta_1X_1 + \beta_2X_2 + \beta_4X_4 + \beta_5X_5 + u$
M29	$Y_{29} = \beta_0 + \beta_1X_1 + \beta_3X_3 + \beta_4X_4 + \beta_5X_5 + u$
M30	$Y_{30} = \beta_0 + \beta_2X_2 + \beta_3X_3 + \beta_4X_4 + \beta_5X_5 + u$
M31	$Y_{31} = \beta_0 + \beta_1X_1 + \beta_2X_2 + \beta_3X_3 + \beta_4X_4 + \beta_5X_5 + u$
Models With First Order Interactions	
M32	$Y_{32} = \beta_0 + \beta_1X_1 + \beta_2X_2 + \beta_{12}X_{12} + u$
M33	$Y_{33} = \beta_0 + \beta_1X_1 + \beta_3X_3 + \beta_{13}X_{13} + u$
M34	$Y_{34} = \beta_0 + \beta_1X_1 + \beta_4X_4 + \beta_{14}X_{14} + u$
M35	$Y_{35} = \beta_0 + \beta_1X_1 + \beta_5X_5 + \beta_{15}X_{15} + u$
M36	$Y_{36} = \beta_0 + \beta_2X_2 + \beta_3X_3 + \beta_{23}X_{23} + u$
M37	$Y_{37} = \beta_0 + \beta_2X_2 + \beta_4X_4 + \beta_{24}X_{24} + u$
M38	$Y_{38} = \beta_0 + \beta_2X_2 + \beta_5X_5 + \beta_{25}X_{25} + u$
M39	$Y_{39} = \beta_0 + \beta_3X_3 + \beta_4X_4 + \beta_{34}X_{34} + u$
M40	$Y_{40} = \beta_0 + \beta_3X_3 + \beta_5X_5 + \beta_{35}X_{35} + u$
M41	$Y_{41} = \beta_0 + \beta_4X_4 + \beta_5X_5 + \beta_{45}X_{45} + u$
M42	$Y_{42} = \beta_0 + \beta_1X_1 + \beta_2X_2 + \beta_3X_3 + \beta_{12}X_{12} + \beta_{13}X_{13} + \beta_{23}X_{23} + u$
:	:
M57	$Y_{57} = \beta_0 + \beta_1X_1 + \beta_2X_2 + \beta_3X_3 + \beta_4X_4 + \beta_5X_5 + \beta_{12}X_{12} + \beta_{13}X_{13} + \beta_{14}X_{14} + \beta_{15}X_{15} + \beta_{23}X_{23} + \beta_{24}X_{24} + \beta_{25}X_{25} + \beta_{34}X_{34} + \beta_{35}X_{35} + \beta_{45}X_{45} + u$
Models With Second Order Interactions	
M58	$Y_{58} = \beta_0 + \beta_1X_1 + \beta_2X_2 + \beta_3X_3 + \beta_{12}X_{12} + \beta_{13}X_{13} + \beta_{23}X_{23} + \beta_{123}X_{123} + u$
M59	$Y_{59} = \beta_0 + \beta_1X_1 + \beta_2X_2 + \beta_4X_4 + \beta_{12}X_{12} + \beta_{14}X_{14} + \beta_{24}X_{24} + \beta_{124}X_{124} + u$
M60	$Y_{60} = \beta_0 + \beta_1X_1 + \beta_2X_2 + \beta_5X_5 + \beta_{12}X_{12} + \beta_{15}X_{15} + \beta_{25}X_{25} + \beta_{125}X_{125} + u$
:	:
M73	$Y_{73} = \beta_0 + \beta_1X_1 + \beta_2X_2 + \beta_3X_3 + \beta_4X_4 + \beta_5X_5 + \beta_{12}X_{12} + \beta_{13}X_{13} + \beta_{14}X_{14} + \beta_{15}X_{15} + \beta_{23}X_{23} + \beta_{24}X_{24} + \beta_{25}X_{25} + \beta_{34}X_{34} + \beta_{35}X_{35} + \beta_{45}X_{45} + \beta_{123}X_{123} + \beta_{124}X_{124} + \beta_{125}X_{125} + \beta_{134}X_{134} + \beta_{135}X_{135} + \beta_{234}X_{234} + \beta_{235}X_{235} + \beta_{345}X_{345} + u$
Models With Third Order Interactions	
M74	$Y_{74} = \beta_0 + \beta_1X_1 + \beta_2X_2 + \beta_3X_3 + \beta_4X_4 + \beta_{12}X_{12} + \beta_{13}X_{13} + \beta_{14}X_{14} + \beta_{23}X_{23} + \beta_{24}X_{24} + \beta_{34}X_{34} + \beta_{123}X_{123} + \beta_{124}X_{124} + \beta_{134}X_{134} + \beta_{234}X_{234} + \beta_{1234}X_{1234} + u$
:	:
M79	$Y_{79} = \beta_0 + \beta_1X_1 + \beta_2X_2 + \beta_3X_3 + \beta_4X_4 + \beta_5X_5 + \beta_{12}X_{12} + \beta_{13}X_{13} + \beta_{14}X_{14} + \beta_{15}X_{15} + \beta_{23}X_{23} + \beta_{24}X_{24} + \beta_{25}X_{25} + \beta_{34}X_{34} + \beta_{35}X_{35} + \beta_{45}X_{45} + \beta_{123}X_{123} + \beta_{124}X_{124} + \beta_{125}X_{125} + \beta_{134}X_{134} + \beta_{135}X_{135} + \beta_{145}X_{145} + \beta_{234}X_{234} + \beta_{235}X_{235} + \beta_{245}X_{245} + \beta_{345}X_{345} + \beta_{1234}X_{1234} + \beta_{1235}X_{1235} + \beta_{1245}X_{1245} + \beta_{1345}X_{1345} + \beta_{2345}X_{2345} + u$
Models With Fourth Order Interactions	
M80	$Y_{80} = \beta_0 + \beta_1X_1 + \beta_2X_2 + \beta_3X_3 + \beta_4X_4 + \beta_5X_5 + \beta_{12}X_{12} + \beta_{13}X_{13} + \beta_{14}X_{14} + \beta_{15}X_{15} + \beta_{23}X_{23} + \beta_{24}X_{24} + \beta_{25}X_{25} + \beta_{34}X_{34} + \beta_{35}X_{35} + \beta_{45}X_{45} + \beta_{123}X_{123} + \beta_{124}X_{124} + \beta_{125}X_{125} + \beta_{134}X_{134} + \beta_{135}X_{135} + \beta_{145}X_{145} + \beta_{234}X_{234} + \beta_{235}X_{235} + \beta_{245}X_{245} + \beta_{345}X_{345} + \beta_{1234}X_{1234} + \beta_{1235}X_{1235} + \beta_{1245}X_{1245} + \beta_{1345}X_{1345} + \beta_{2345}X_{2345} + \beta_{12345}X_{12345} + u$